RESEARCH DEPARTMENT

The design of gradual transition absorbers for a free-field room

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THE DESIGN OF GRADUAL TRANSITION ABSORBERS FOR A FREE-FIELD ROOM

Research Report No. L-057

(1965/4)

D. Maurice

J.W. Head, M.A., F.Inst.P., A.M.I.E.E., F.I.M.A.

for Head of Research Department

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SUMMARY

The acoustic treatment of free-field rooms usually takes the form of projecting wedges of sound absorbing material. Measurements of the reflexion coefficient of such wedges, made at normal incidence in a travelling wave duct, show that it is possible to achieve a reflexion coefficient R less than 0·1 when the ratio of the length l of the wedges to the acoustic wavelength λ is not less than about 0·2. Existing theories of the absorbent properties of wedges are unsatisfactory since they predict that l/λ has to exceed about 0·4 before R is consistently less than 0·1, and all constants of the material are assumed independent of frequency. A technique is here described whereby theoretical values comparable with the above-mentioned experimental results can be obtained by allowing for the mechanical properties of the material. This technique involves the multiplication of a number of matrices each having two rows and columns, and unit determinant. This can be done by means of an electronic computer programme in which several adjustable parameters can be fed in initially by means of a data-tape. A particular set of values of these parameters (Equation (13)) is such that the calculated $R < 0{\cdot}1$ for $l/\lambda > 0{\cdot}2$. Other sets of values indicate that theoretically R can be less than 0·1 for l/λ above 0·1; this is better than has so far been obtained in practice. Further calculations could be made with little or no modification of the existing computer programme for many other such sets. At present, however, there is insufficient information on the actual variation of compressibility and density within the wedges to determine what values of the parameters are physically acceptable. This information is badly needed before further calculations can be profitable.

1. INTRODUCTION

For certain acoustic measurements it is necessary to simulate free-field conditions indoors by providing a room in which all surfaces are lined with highly absorbing material to reduce reflexions. Various forms of lining have been tried at different times and the least reflexion is obtained when the acoustic treatment is composed of tapered absorbers, usually in the form of wedges, projecting from the surfaces. In the low-frequency range, in which it is most difficult to obtain adequate absorption, the wedges can be represented as a medium whose constants change gradually from those of air at the wedge tip to those of the continuous material at the base of the wedges. Such absorbers are therefore called ‘gradual transition absorbers’.

Measurements of reflexion coefficient R made on this type of treatment at normal incidence in a travelling wave duct show that R can be consistently below 0·1 down to a frequency at which the ratio of the length l of the wedges to the acoustic wavelength λ is about 0·2. Below this frequency the reflexion coefficient rises sharply. Until recently wedges were constructed of glass fibre or mineral wool, materials in which the absorption is mainly due to the flow resistance although there is some evidence[1] that frictional loss associated with movement of the fibres is also important. In several new free-field rooms, however, expanded polyurethane has been used. With this material[2] absorption at low frequencies is mainly due to losses associated with the mechanical constants, and the flow resistance is relatively less important.

Several workers have investigated the absorbent properties of wedges theoretically, but most have assumed that only the flow resistance was important, and have predicted a minimum value of l/λ (to obtain $R < 0{\cdot}1$ consistently) of about 0·4. Fig. 1, taken from reference 2, shows the effect of varying the length of taper on the normal-incidence reflexion characteristic. If, as most workers have assumed, all constants of the material were independent of frequency, the curves of Fig. 1 would have been identical except for a displacement parallel to the frequency axis.

This report describes a further attempt to examine theoretically sound absorption by wedges, taking into account the mechanical constants, including isothermal or other losses which vary with frequency. For this purpose it is first necessary to make assumptions about the way in which the compressibility and density vary with distance along the wedge. These assumptions must involve a number of parameters which can be fed into a computer programme, so that the effect of varying these parameters can be investigated. Such a programme was written by Mr. R.W. Lee for the Elliott 803 computer at present used by the BBC. This programme could also be run on an Elliott 503. It is not easy to obtain reliable values of the compressibility and density of absorbing materials, and part of the object of this report is to put forward tentative values of these quantities which can be confirmed or amended in the light of later investigations which may not be directly related to absorption or acoustic properties.

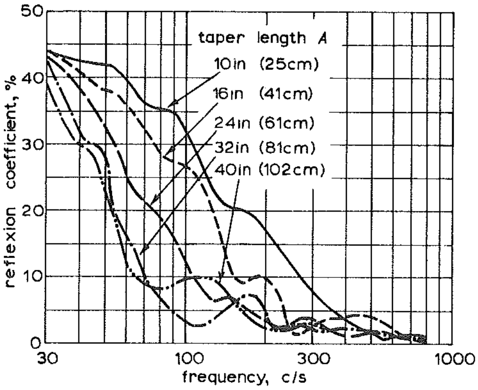

Fig. 1 - Variation of reflexion coefficient with length of wedge taper

No attempt is made here to relate these quantities to measurable data associated with the shape and material of the wedges. This relation, however, will be an important part of the task of anyone seeking to determine whether the variations of reflexion coefficient calculated below correspond to physically realizable wedges.

The essential element in the computer programme is the multiplication of a number of matrices each having two rows and columns and unit determinant. Any improved determination of the law of variation of compressibility and density might affect the determination of the elements of these matrices, but not the handling of the matrices once their elements were determined.

As Walther[3] has pointed out, the differential equations applicable to electromagnetic and acoustic inhomogeneous absorbers are very closely related, so that the present investigation is also relevant, for example, to the study of cables whose electrical parameters vary with distance.

Although the basic equations used here are the same as those of Walther, they have been manipulated in a different way, taking advantage of matrix techniques, as explained in Section 2. Results are discussed in Section 3.

2. THE THEORETICAL BEHAVIOUR OF A GRADUAL TRANSITION ABSORBER

The basic equations governing the propagation of sound through an absorbing medium are:

$$\frac{\mathrm{d}v}{\mathrm{d}x} = -\,\mathrm{j}\omega\kappa p \tag{1}$$

$$\frac{\mathrm{d}p}{\mathrm{d}x} = -\,\mathrm{j}\omega\rho v \tag{2}$$

where x is the distance along the medium from a fixed point, p is the sound pressure and v the sound velocity at distance x, and κ, ρ are respectively the compressibility and density at distance x, and $\omega/2\pi$ is the frequency.

We follow Walther in assuming that a plane sound wave at frequency $\omega/2\pi$ is propagated parallel to the x-axis, and that κ, ρ, p and v depend only on x; the boundary conditions are that when $x = 0$, there is a rigid terminating wall as in Fig. 2, and that, for $x > l$, $\rho(x) = \rho_0$ and $\kappa(x) = \kappa_0$, values appropriate to the homogeneous medium in front of the absorber. Our general method of attack will be to divide the absorbing panel into a number n of slabs, within each of which we shall treat $\kappa(x)$ and $\rho(x)$ as constant and having the value appropriate to the mid-point of the slab; at the boundary between consecutive slabs we assume that p and v are continuous (but not their derivatives). This differs from the general treatment of Walther[3], who derived directly from (1) and (2) a non-linear Riccati-type differential equation for the reflexion coefficient. For the numerical calculations so far undertaken, n has been taken as 10.

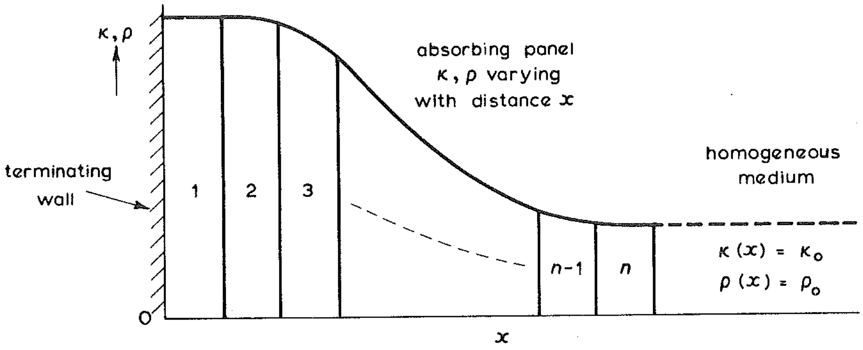

Fig. 2 - General behaviour of a gradual transition absorber

The assumed variation of ρ, κ with x is of the form

$$\rho(x) = \left[A^{(1-x/l)} - \mathrm{j}\left(B + \frac{C}{l/\lambda}\right)(D^{(1-x/l)} - 1)\right]\rho_0 \tag{3}$$

$$\kappa(x) = \left[D^{(1-x/l)} - \mathrm{j}E(A^{(1-x/l)} - 1)\right]\kappa_0 \tag{4}$$

and the computer programme is so arranged that A, B, C, D and E can be fed in initially on a data tape. If the exponential variation of the real and imaginary parts of ρ and κ with x is later considered not to be the most appropriate, only a small part of the programme would have to be changed. From equations (3) and (4) it is seen that $\rho = \rho_0$ and $\kappa = \kappa_0$ inside the absorbing medium when $x = l$, whatever the values of A, B, C, D and E may be.

If within any one slab κ and ρ are treated as constant, we find from (1) and (2) that p and v both satisfy

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\omega^2\kappa\rho y \tag{5}$$

Hence in the homogenous medium outside the absorber, the acoustic wavelength λ is given by

$$\frac{2\pi}{\lambda} = \omega(\kappa_0\rho_0)^{1/2} \tag{6}$$

Within the absorbing medium, let

$$\alpha = \mathrm{j}\omega(\kappa\rho)^{1/2} = \mathrm{j}\frac{2\pi}{\lambda}\left(\frac{\kappa\rho}{\kappa_0\rho_0}\right)^{1/2} \tag{7}$$

and consider a particular slab extending from $x = x_0$ to $x = x_0 + (l/n)$. Let p_1, v_1 be the sound pressure and velocity when $x = x_0$ and p_2, v_2 be the sound pressure and velocity when $x = x_0 + l/n$. Then it can be shown that

$$\begin{pmatrix} p_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} \cosh(\alpha l/n) & -(\rho/\kappa)^{1/2}\sinh(\alpha l/n) \\ -(\kappa/\rho)^{1/2}\sinh(\alpha l/n) & \cosh(\alpha l/n) \end{pmatrix} \begin{pmatrix} p_1 \\ v_1 \end{pmatrix} \tag{8}$$

Equation (8) follows from the fact that within the slab both p and v are of the form

$$A\mathrm{e}^{\alpha x} + B\mathrm{e}^{-\alpha x}$$

and that p and v have known values at both ends of the slab and satisfy equations (1) and (2), but the matrix form of (8) means that we can immediately write down the sound pressure p_n and velocity v_n at the end of the absorbing medium (when $x = l$)

in terms of the sound pressure and velocity at the terminating wall (where the velocity is zero and the pressure can be taken as unity). We thus obtain

$$\begin{pmatrix} p_n \\ v_n \end{pmatrix} = \prod_{r=n}^{1} \begin{pmatrix} \cosh (\alpha_r l/n) & -(\rho_r/\kappa_r)^{\frac{1}{2}} \sinh (\alpha_r l/n) \\ -(\kappa_r/\rho_r)^{\frac{1}{2}} \sinh (\alpha_r l/n) & \cosh (\alpha_r l/n) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (9)$$

For numerical calculation, the unit of v should be $(\kappa_0/\rho_0)^{\frac{1}{2}}$ times the unit of p, and all calculation can then be done in (9) with relative values of ρ and κ at the mid-point of each slab, obtained by omitting the factor ρ_0 on the right hand side of (3) and the factor κ_0 on the right hand side of (4). The reflexion coefficient R is then given by

$$R = \frac{-(p_n/V_n) - (\rho_0/\kappa_0)^{\frac{1}{2}}}{-(p_n/V_n) + (\rho_0/\kappa_0)^{\frac{1}{2}}} \qquad (10)$$

The minus sign preceding (p_n/V_n) is necessary, as Walther points out, because of the choice of co-ordinates in Fig. 1, and in (10) all factors involving ρ_0 and κ_0 cancel provided that relative values of κ and ρ are used in the numerical calculations, as already noted.

If α_r, ρ_r and κ_r are constant with r, Wilson[4] has shown that (9) simplifies to

$$\begin{pmatrix} p_n \\ v_n \end{pmatrix} = \begin{pmatrix} \cosh \alpha l & -(\rho/\kappa)^{\frac{1}{2}} \sinh \alpha l \\ -(\kappa/\rho)^{\frac{1}{2}} \sinh \alpha l & \cosh \alpha l \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (11)$$

but as in our case α_r etc. are not constant with r, there is no alternative to performing numerically the matrix multiplications specified in (10). This formidable task for a human being, however, is easily done by means of an electronic computer. Such calculations have therefore been carried out on the BBC's Elliott 803 computer for various values of the parameters in equations (3) and (4), taking n as 10. In principle, there is no reason why n should not be increased, but at this exploratory stage, 10 was regarded as a sufficiently high value. Results are discussed in Section 3.

It should be noted that in (9) we only need to be able to determine κ_r and ρ_r (and hence α_r) at the mid-point of each slab. If therefore it was later found that some law of variation of κ and ρ with x other than the exponential variation assumed in equations (3) and (4) were preferable, the computer programme could be adjusted relatively easily. In the absence of adequate information, however, the exponential variation suggested by Walther has been accepted provisionally.

3. RESULTS OBTAINED FROM PARTICULAR CASES OF EQUATIONS (3), (4) AND (9)

Walther discusses results for the case [in the notation of (3) and (4)]

$$A = 3{\cdot}3 \quad B = 1{\cdot}5 \quad C = 0 \quad D = 50 \quad E = 0{\cdot}3 \qquad (12)$$

and the present investigation was begun with the idea of assessing the effect of a small value of C different from zero. C was therefore initially taken as 0·2, and at first D was the parameter mainly varied. For large values of D, due allowance in the computer programme was made for the fact that the incident radiation might fail to penetrate some of the slabs nearest the terminating wall.

For large and moderate values of D, the reflexion coefficient R does not fall below the desired value 0·1 until l/λ greatly exceeds 0·2, whereas experimentally R was consistently below 0·1 for $l/\lambda \geqslant 0{\cdot}2$. When A and D were both in the region 2 - 2·5, however, a more desirable type of curve for R as a function of l/λ was obtained; these curves had a general tendency for R to fall below 0·1 when l/λ was in the region of 0·2, but subsequently to rise to a value in the neighbourhood of 0·13 when l/λ was about 0·4. No alteration of A and D in this region appeared to have a significant effect. Alteration of C to 0·5 (instead of 0·2) appeared to have an adverse effect, and it was not until E was increased from 0·3 to 1 (with $A = D = 2{\cdot}2$, $B = 1{\cdot}5$ and $C = 0{\cdot}2$) that for the first time a calculated curve was obtained which genuinely met the specification that R should not exceed 0·1 for $l/\lambda \geqslant 0{\cdot}2$. This curve is plotted in Fig. 3. R is necessarily 1 for $l/\lambda = 0$, and was calculated for $l/\lambda = 0{\cdot}1$ and 0·2 (0·05) 0·7.

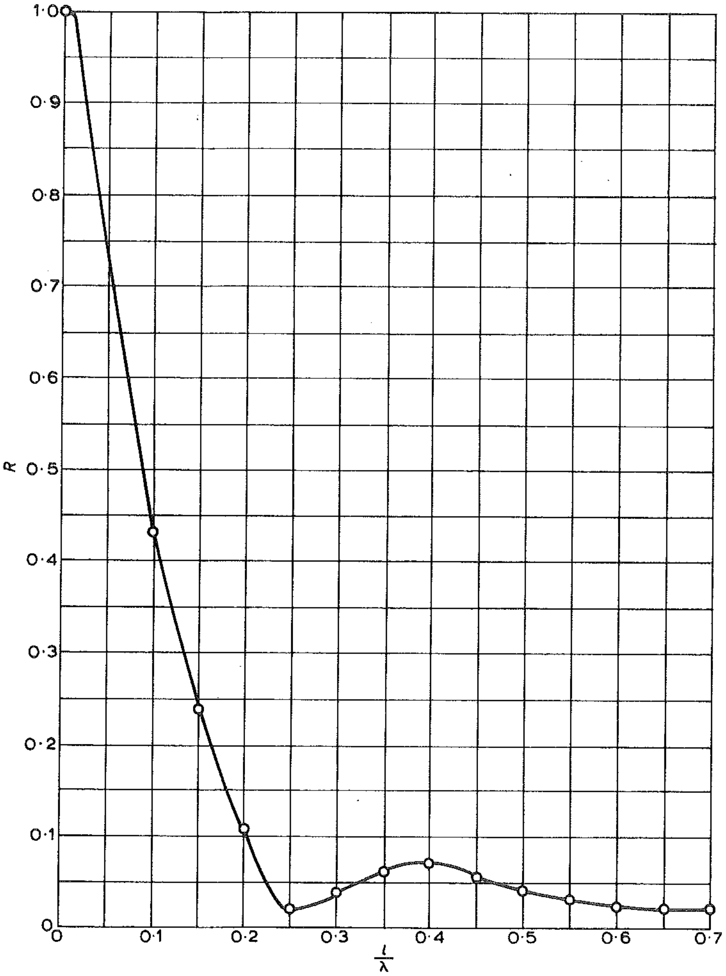

Fig. 3 - Variation of R with l/λ for the parameter values specified in equation (13)

4. CONCLUSIONS

The foregoing investigation indicates that, mathematically, the region near

$$A = D = 2{\cdot}2 \quad B = 1 \quad C = 0{\cdot}2 \quad E = 1 \tag{13}$$

is one in which calculated curves of R against l/λ meeting the requirement $R < 0{\cdot}1$ for $l/\lambda > 0{\cdot}2$ can be found. It does not indicate that this is the only such region, or that these values adequately represent the actual behaviour of the material.

If

$$A = 2{\cdot}2 \quad B = 3{\cdot}5 \quad C = 0{\cdot}2 \quad D = 2{\cdot}2 \text{ and } E = 1{\cdot}5 \tag{14}$$

the situation is theoretically even better, for R is in this case below $0{\cdot}1$ for $l/\lambda > 0{\cdot}1$ only. It should be noted that this result is appreciably better than has as yet been obtained in practice.

The values (13) are therefore put forward tentatively as appropriate for substitution into (3) and (4) to give a possible description of the behaviour of the absorbing material in question. Confirmation or condemnation of these values, especially by those interested in the behaviour of κ and ρ for reasons other than acoustics, is now required. This would narrow down the field of possible sets of values of A, B, C, D and E (or alternative parameters) within which further investigation by means of the computer could usefully be undertaken.

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